$\Delta p/\Delta p_0 = 0.605, 0.528$. This results in Δp's of 1.64, 0.996, and 0.870 psf for the asymptotic, constant overpressure, and rising pressure cases. For comparison minimizing the front shock only would give $\Delta p_F = 0.958$ and 0.645 psf for $J = 0$, 1. Thus as was already evident from the discussion, only the $J = 1$ case differs significantly from the Δp_F only optimization. If the flight altitude is decreased to 40,000 ft, m becomes 4.68×10^{-3} and $W = 0.203$. In this case the Δp's are 2.26, 0.967, and 0.464 psf for the asymptotic, $J = 0$ and $J = 1$ cases. Thus, as previously pointed out in Ref. 1, the use of midfield optimizations can nearly eliminate the effect of increased flight altitude. This allows lower flight altitudes with comparable or reduced sonic boom strengths. A bangless boom can be obtained for 40,000 ft altitude if the present 300 ft aircraft weighs less than 355,000 lb. This weight, although stringent, is not beyond the realm of possibility.

As discussed in Refs. 1 and 2, the effective cross-sectional areas required for these shapes are not extreme and the present shapes are more easily approached than the asymptotic case. The discontinuity in $\mathfrak{F}$ at Y_C requires only a $(Y - Y_C)^{3/2}$ or $(Y - Y_C)^{1/2}$ effective area or lift distribution, respectively, and should be easy to approximate in a real design.

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Delta Wing Shock Shape at Hypersonic Speed

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THE considerable literature on delta wing hypersonic aerodynamics offers little information on the three-dimensional shock shape. The situation appears anomalous in view of the fact that the shock shape is the starting point and the controlling feature in many theoretical solutions of the delta wing flowfield. The ability to calculate the shock envelope is of practical importance when, for example, the effects of possible interaction with other vehicle components are considered, which can lead to serious heating, stability, and control problems.

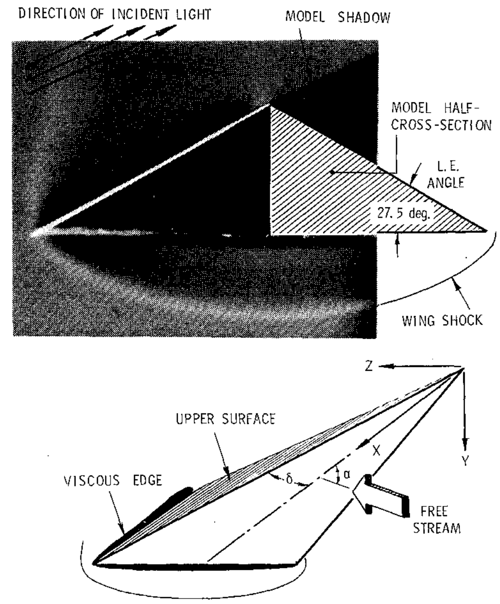

Fig. 1 Typical vapor screen photograph ($\alpha = 13°$); coordinate system and angles.

Experimentally, the determination of the three-dimensional shock surface is usually a tedious point-by-point procedure involving pitot-tube traversing of the flowfield. During a recent program of flow-visualization tests on delta wings using the vapor screen method (commonly employed for qualitative flowfield studies in wind tunnels) good quality photographs were obtained in which the entire shock envelope cross section was clearly visible (see Fig. 1). This suggested the use of the vapor screen technique as a rapid means of determining the shock envelope of complex shapes. However, due to unknown aerodynamic effects in a condensing flow, the quantitative validity of the method has first to be established.

The present Note is concerned with an exploratory evaluation of the above idea. A comparison of the shock measurements with schlieren results as well as with theoretical calculations by the method of Squire[1] has been made. The cases studied herein correspond to shocks detached from the leading edges, a condition typical of hypersonic flight.

A flat-bottom delta wing model with "roof-top" and sharp leading-edges swept back at 75° was tested in the Langley 11-in. Blowdown Tunnel fitted with a two-dimensional $M = 6.8$ nozzle, using dry air. The tunnel was run "cold," i.e., at ambient stagnation temperature, to permit a partial condensation in the flow.† The freestream Reynolds number was approximately 0.4 million per in., giving a model length Reynolds number of nearly 4 million. The vapor screen was formed by illuminating a thin slice (0.1 in.) of the flow normal to the freestream and the bottom surface of the model, positioned about 1 in. upstream of the trailing-edge. Flowfield photographs were taken at incidence angle of the lower surface $\alpha = 8°, 13°$, and $18°$.

Schlieren photographs were obtained in subsequent hot-flow runs repeated at the same incidence angles (with model Reynolds number of about 4.5 million). This provided center-plane shock data for comparison with the vapor screen results and, as seen in Fig. 2, excellent agreement is found.

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† It may be noted that while at lower Mach numbers the vapor screen depends on a two-phase flow containing water vapor, in this case the liquefaction of oxygen, or even a three-phase flow with solid carbon dioxide particles, is likely to occur.

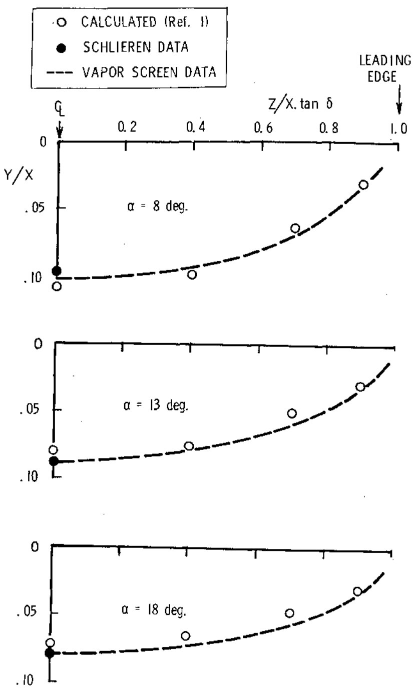

Fig. 2 Comparison of calculated and measured shock data.

It is expected that such agreement in the shock position between hot and cold tests would persist at points off the center plane.

The shock position in the center-plane and at several spanwise locations was calculated from the charts presented by Squire[1] for Mach 6.8 and ratio of specific heats 1.4. The computed points are compared with the shock shape traced off the vapor screen photographs, corrected for perspective distortion, in Fig. 2. Here again a generally good agreement is found in the range of incidence investigated.

The use of Squire's theory to check the experimental results requires comment. This theory has not been previously assessed experimentally for shock shape in published literature (except in the plane of symmetry by optical measurements, as in Ref. 2 where agreement similar to that found here was obtained at $M = 8.2$). However, it has been found to yield satisfactory predictions of spanwise pressure distribution over a variety of delta wing shapes in the Mach number range 2.47 to 8.6.[1] Squire's solution for the wing shock shape was used to calculate the interaction with the flap shock on a delta wing with trailing-edge flap at $M = 8.2$[3] and good agreement with experiment was obtained. Thus, it is felt that Squire's theory provides the shock shape for delta wings with reasonable accuracy.

While the data presented here is limited to one Mach number and model configuration, the schlieren results as well as theoretical calculations of this study suggest that the vapor screen technique is useful for quantitative shock information. A very simple experimental technique would then appear to be available for determining the hypersonic shock envelope of complex vehicle configurations.

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A Comparison of Some Finite Element and Finite Difference Methods for a Simple Sloshing Problem

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A. Introduction

By a finite element method, we mean an application of a variational principle[1] and a quadrature formula for integration over each element. By a finite difference method, we mean the use of finite difference formulas derivable from a truncated Taylor series at the nodes. In the former method, equations at the nodes are obtained by "minimizing" the integral with respect to each independent generalized coordinate at the nodes.

Recently, Hunt[2] employed a second-order finite element formula which is derivable from Hamilton's principle, subject to a zero divergence condition (i.e., incompressible flow). A trapezoidal rule was implicitly used. Hunt's method is ingenious because only one normal displacement coordinate is required at each midpoint of the sides of the square element. But, before extending it to more general cases, it seems that a comparison of Hunt's method to other methods of computing fuel sloshing for a two-dimensional rectangular tank is worthwhile. Computation based on Hunt's method and two other methods will be given in this paper.

For heat conduction problems, analogous comparisons are given by Emery and Carson.[3] The subject of the present paper, however, is concerned with potential flow with a free surface.

B. Description of Methods

1. *Hunt's method (method I)*

For each element, there is a constraint of zero divergence, which therefore generates one dependent generalized coordinate and reduces the independent coordinates by one. From Hamilton's principle and trapezoidal rule, one finds (see Fig. 1).

$$2\bar{L} = \sum_{e} \tfrac{1}{2}\rho\omega^2 h^2[\zeta_+{}^2 + \zeta_-{}^2 + \xi_+{}^2 + \xi_-{}^2] \qquad (1)$$

which is to be minimized subject to the constraint condition that

$$\zeta_+ - \zeta_- + \xi_+ - \xi_- = 0 \qquad (2)$$

This process leads to an eigenvalue problem, where the eigenvalue is related to the natural frequency parameter and each eigenvector indirectly determines the respective free-surface mode shape.

2. *A simple finite difference method (method II)*

The governing differential equation is the well-known Laplace equation. The difference operator which is used at

Fig. 1 Generalized displacements of a square element; Hunt's method.

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